

Successive transitions and intermediate chiral phase in a superfluid ^3He film

Hikaru Kawamura

Faculty of Engineering and Design, Kyoto Institute of Technology, Kyoto 606-8585, Japan

Superfluidity ordering of thin ^3He films is studied by Monte Carlo simulations based on a two-dimensional lattice spin model with $Z_2 \times U(1) \times SO(3)$ symmetry. Successive phase transitions with an intermediate ‘chiral’ phase, in which the \mathbf{l} -vector aligns keeping the phase of the condensate disordered, is found. Possible experiments to detect the successive transitions are discussed.

67.70.+n, 67.57.Lm

It is now well established that the superfluidity transition of ^4He films belongs to the Kosterlitz-Thouless (KT) universality class governing a variety of two-dimensional (2D) systems with $U(1)$ symmetry [1]. By contrast, the nature of the 2D phase transition of its isotope, *i.e.*, the superfluidity transition of ^3He films of thickness of order the zero-temperature coherence length, $0.01\mu\text{m} \lesssim \xi_0 \lesssim 0.05\mu\text{m}$, has not been well understood. Although the complex structure in the order parameter of ^3He [2] makes its transition behavior an intriguing problem, the experimental observation of the possible nontrivial critical behavior of *bulk* ^3He is difficult, since, due to its long coherence length, the critical regime is limited to a narrow temperature range close to T_c , *i.e.*, $t \equiv |(T - T_c)/T_c| \lesssim t^* \approx 10^{-6}$ [3]. By contrast, in the case of thin films, enhanced effects of fluctuations due to the low dimensionality give rise to much wider critical regime, as wide as $t^* \approx 10^{-1} \sim 10^{-2}$ [4]. Hence, there appears to be a good possibility to experimentally observe the novel transition behavior originated from the complex order parameter.

There have been many experimental works on the superfluidity ordering of ^3He in restricted geometries including cylindrical pores [5,6] and thin flat films [7,8]. Indeed, several experiments reported the observation of novel ordering behaviors, even a signature of new phase [6,7]. Experimentally, one challenging problem is to make measurements on a sample which can be regarded as a good 2D system, *i.e.*, a thin flat film of uniform thickness of order $0.1\mu\text{m}$ or less on sufficiently specular boundaries. For such an ideal 2D system, theoretical calculation suggests that the stable superfluidity state is that of the A phase down to zero pressure [9], which has also been confirmed experimentally [8]. The order parameter can be described as a triad of three orthogonal unit vectors \mathbf{m} , \mathbf{n} and $\mathbf{l} = \mathbf{m} \times \mathbf{n}$, where the \mathbf{l} -vector is kept perpendicular to the surface.

Some time ago, Stein and Cross argued that such superfluidity state with the A -phase structure possessed the novel twofold Ising-like degeneracy, according as the \mathbf{l} -vector was either *parallel* or *antiparallel* with the surface normal [10]. This Z_2 symmetry coexists with the standard $U(1)$ gauge symmetry associated with the phase of the condensate. The latter symmetry corresponds to a

continuous rotation of the \mathbf{m} - and \mathbf{n} -vectors around the \mathbf{l} -vector. Stein and Cross then argued that this novel Z_2 symmetry might lead to an Ising-like transition which might ‘compete’ with the standard KT transition associated with the $U(1)$ gauge symmetry, although the detailed nature of the transition was not specified.

Since then, 2D phase transitions associated with the $Z_2 \times U(1)$ symmetry have been studied in a different area, *i.e.*, in the context of frustrated 2D XY systems such as the triangular-lattice XY antiferromagnet [11] or the Josephson-junction array in a magnetic field [12]. In these problems, the Z_2 Ising-like degree of freedom has been called ‘chirality’ [13]. While these studies have established the occurrence of a phase transition with a sharp specific-heat anomaly, there still remains some controversy as regards whether the $Z(2)$ and $U(1)$ degrees of freedom order simultaneously, or at two close but distinct temperatures. Meanwhile, some differences exist between these frustrated XY systems and ^3He films. Particularly important point neglected in the previous analysis [10] may be the fact that ^3He has an additional internal symmetry, $SO(3)$, associated with the nuclear spin degree of freedom of the condensate.

In the present Letter, I perform a numerical study of the superfluidity ordering of ^3He films in order to clarify possible thermodynamic phases and the transition behavior between them. For that purpose, I introduce a 2D lattice spin (pseudospin) model which possesses the expected full symmetry of the order parameter, $Z_2 \times U(1) \times SO(3)$, and perform Monte Carlo simulations. It is found that the model exhibits two successive transitions with a ‘chiral’ intermediate phase.

The order parameter describing the 2D superfluidity of ^3He may be described by a 3×2 tensor variable $A_{\mu j}$,

$$A_{\mu j} = Ad_{\mu}(m_j + in_j), \quad |\mathbf{d}| = |\mathbf{m}| = |\mathbf{n}| = 1, \quad \mathbf{m} \cdot \mathbf{n} = 0, \quad (1)$$

where $\mu = x, y, z$ refers to the spin component, $j = x, y$ refers to the real-space coordinate (the film surface is taken to be the xy -plane), \mathbf{d} is a three-component unit vector in spin space representing the spin state of the condensate, while \mathbf{m} and \mathbf{n} are mutually orthogonal two-component unit vectors in real space representing the orbital state of the condensate. The Ising-like variable, or

the chirality, is defined by $\tau = \text{sign}(l_z) = m_x n_y - m_y n_x$, where $\tau = \pm 1$ represents the 1-vector pointing either up or down along the surface normal.

In deriving an appropriate model Hamiltonian, I start with the standard Ginzburg-Landau free energy in the London limit,

$$\mathcal{H}_{GL} \approx K(|\nabla \mathbf{A}_\mu|^2 + \delta |\text{div} \mathbf{A}_\mu|^2), \quad (2)$$

where $(\mathbf{A}_\mu)_i = A_{\mu i}$. Although there generally exists a spatially-anisotropic gradient term (the second term of Eq.(2)), renormalization-group $\epsilon = 4 - d$ expansion analysis showed that such spatial anisotropy was irrelevant at the superfluidity transition, *i.e.*, $\delta \rightarrow 0$ upon renormalization [3]. Hence, for simplicity, I drop the second term here. Neglecting the weak dipolar interaction of order $10^{-1} \mu\text{K}$, restricting the space to 2D, parametrizing the \mathbf{m} -vector as $\mathbf{m} = (\cos \theta, -\tau \sin \theta)$ where $0 \leq \theta < 2\pi$ is the phase angle of the condensate, and discretizing the continuum into the lattice, one obtains

$$\mathcal{H} = -J \sum_{\langle ij \rangle} (1 + \tau_i \tau_j) \cos(\theta_i - \theta_j) \mathbf{d}_i \cdot \mathbf{d}_j, \quad (3)$$

where $J > 0$ is a coupling constant and the summation is taken over all nearest-neighbor pairs on the square lattice. The Hamiltonian (3) can be viewed as a coupled Ising- XY -Heisenberg model with $Z_2 \times U(1) \times SO(3)$ symmetry, in which $\tau_i = \pm 1$ is an Ising variable [Z_2], θ_i being an angle variable [$U(1)$] of the XY -pseudospin $\mathbf{p}_i = (\cos \theta_i, \sin \theta_i)$, and \mathbf{d}_i is a Heisenberg variable [$SO(3)$]. Note that the Hamiltonian (3) also possesses the *local* symmetries under the transformations: (i) $\mathbf{p}_i \rightarrow \tau_i \mathbf{p}_i$; (ii) $\mathbf{d}_i \rightarrow \tau_i \mathbf{d}_i$; (iii) $\mathbf{p}_i \rightarrow \pm \mathbf{p}_i$ and $\mathbf{d}_i \rightarrow \pm \mathbf{d}_i$.

The ordering of the 1-vector, or of the chirality, can be probed via the Ising-magnetization, $\tau = \frac{1}{N} \sum_i \tau_i$ (N is the total number of lattice sites), by calculating the average chirality and the associated Binder ratio,

$$\bar{\tau} = \langle \tau^2 \rangle^{1/2}, \quad g_\tau = \frac{1}{2} \left(3 - \frac{\langle \tau^4 \rangle}{\langle \tau^2 \rangle^2} \right). \quad (4)$$

Since the correlation functions associated with the phase variable θ , or the XY -pseudospin \mathbf{p} , are not invariant under the above local transformations and vanish trivially, one needs to calculate the “local-gauge-invariant” quantity to study the phase ordering. An appropriate quantity is a second-rank symmetric traceless tensor $P_{i\mu\nu} = \frac{1}{\sqrt{2}} (p_{i\mu} p_{i\nu} - \frac{1}{2} \delta_{\mu\nu})$, or equivalently, a new XY -pseudospin variable with the doubled phase angle, $\mathbf{p}'_i = (\cos 2\theta_i, \sin 2\theta_i)$. Via the XY -magnetization, $\mathbf{p}' = \frac{1}{N} \sum_i \mathbf{p}'_i$, the associated Binder ratio is defined by

$$g_\theta = 2 - \frac{\langle \mathbf{p}'^4 \rangle}{\langle \mathbf{p}'^2 \rangle^2}, \quad (5)$$

Likewise, the ordering of the \mathbf{d} -vector is probed via $D_{\mu\nu} = \frac{1}{N} \sum_i D_{i\mu\nu}$, where a second-rank symmetric

traceless tensor is defined by $D_{i\mu\nu} = \frac{1}{\sqrt{3}} (d_{i\mu} d_{i\nu} - \frac{1}{3} \delta_{\mu\nu})$. The corresponding Binder ratio is given by

$$g_d = \frac{1}{2} (7 - 5 \frac{\langle D^4 \rangle}{\langle D^2 \rangle^2}), \quad D^2 \equiv \sum_{\mu\nu} D_{\mu\nu}^2. \quad (6)$$

Monte Carlo simulations are performed based on the standard single-spin-flip Metropolis method. The lattice contains $N = L^2$ sites with $L = 20, 30, 40$ and 60 with periodic boundary conditions. Typically, I generate total of 10^5 Monte Carlo steps per spin (MCS) at each temperature, and $(2.5 \sim 10) \times 10^5$ MCS in the transition region.

The calculated specific heat is shown in Fig.1. A sharp peak is observed at $T/J = 0.754 \pm 0.002$. As is shown in the inset, the peak grows slowly with increasing L , indicative of a continuous transition characterized by the exponent $\alpha \sim 0$.

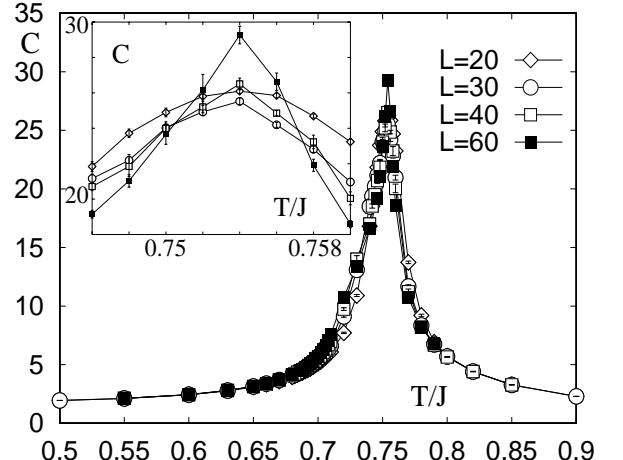


FIG.1 The temperature and size dependence of the specific heat per site. The inset represents a magnified view of the transition region.

The calculated Binder ratios (4)-(6) are shown in Figs.2a-c, respectively. As can be seen from Fig.2a, g_τ for different L cross at a temperature which coincides with the specific-heat-peak temperature. This indicates the occurrence of a continuous transition at $T_{c1}/J = 0.753 \pm 0.001$ into the phase with a finite chiral long-range order, $\bar{\tau} > 0$, accompanied by a sharp specific-heat anomaly. The behavior of $\bar{\tau}$ (not shown here) turns out to be fully consistent with this. By contrast, as can be seen from Figs.2b and c, g_θ and g_d for various L do not cross or merge at T_{c1} , constantly decreasing with increasing L around T_{c1} , a behavior characteristic of a disorder phase. Hence, the state just below T_{c1} is a pure chiral state with only a chiral long-range order while the spin and the spin are kept disordered.

At a lower temperature $T_{c2}/J = 0.692 \pm 0.04$, g_θ for various L merge, and below $T < T_{c2}$, continue to stay on a common curve: See Fig.2b. Such a behavior is

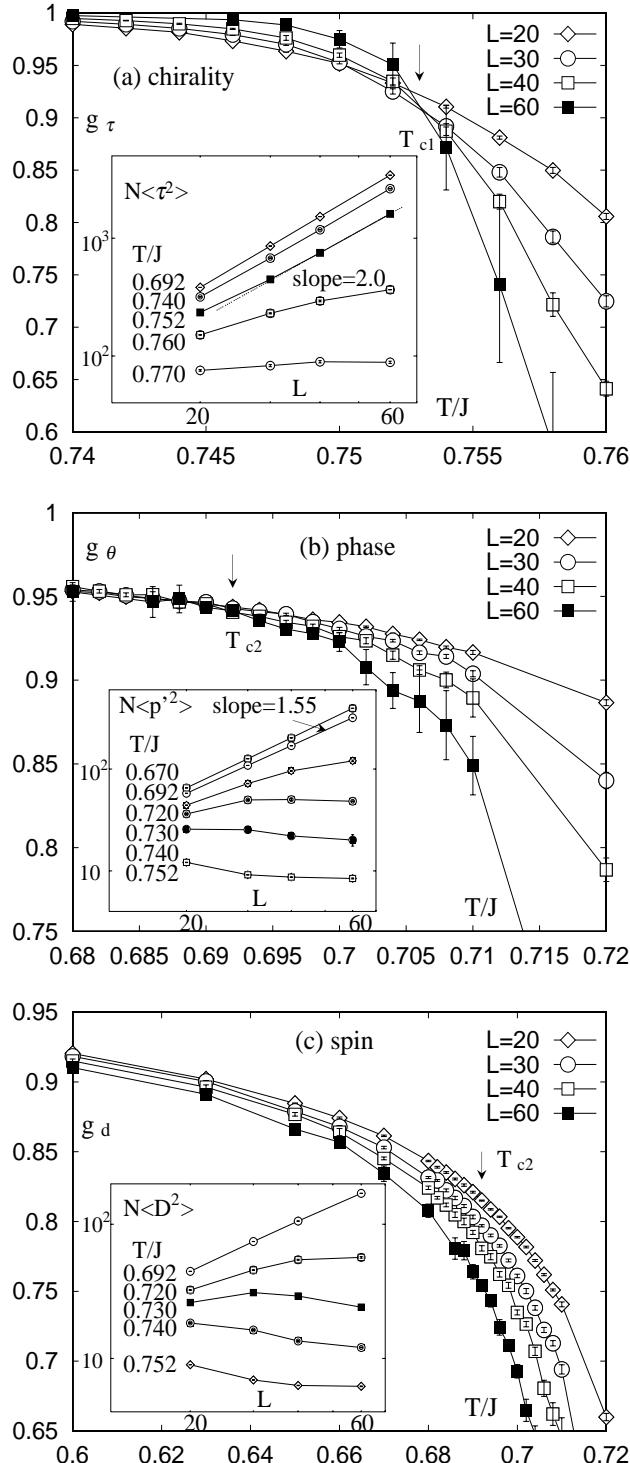


FIG.2 The temperature and size dependence of the Binder ratios in the transition region; (a) of the τ -variable (chirality); (b) of the \mathbf{p}' -variable (phase); (c) of the \mathbf{d} -variable (spin). Note the difference in the temperature scale in each figure. The insets show log-log plots of the L dependence of the ordering susceptibilities, $N < \tau^2 >$ (a), $N < \mathbf{p}'^2 >$ (b) and $N < D^2 >$ (c). In the ordered phase with a nonzero order parameter, the data for larger L should lie on a straight line with a slope equal to two,

while in the disordered phase the data should exhibit a downward curvature staying finite even in the $L \rightarrow \infty$ limit. At criticality and in the KT-like phase, the data should lie on a straight line with a slope $2 - \eta$, which is less than two.

characteristic of the KT-like transition in which the low-temperature phase is a critical phase with algebraically-decaying correlations. Thus, the quasi-long-range order of the $U(1)$ phase is realized below T_{c2} . The absence of any appreciable anomaly in the specific heat around T_{c2} is also consistent with such KT-like transition. By contrast, g_d monotonically decreases with increasing L around T_{c2} , suggesting that the \mathbf{d} -vector remains disordered even at $T < T_{c2}$. This observation is consistent with the fact that the \mathbf{d} -vector is a Heisenberg variable carrying the $SO(3)$ symmetry, and that the 2D Heisenberg model orders only at $T = 0$. I have also checked that the behaviors of the order parameters corroborate the above conclusion. As an example, the L dependence of the ordering susceptibilities, $N < \tau^2 >$, $N < \mathbf{p}'^2 >$ and $N < D^2 >$, are shown on log-log plots in the insets of Fig.2.

Thus, the present model exhibits two successive transitions at $T = T_{c1}$ and T_{c2} , where $T_{c1} \simeq 0.753J > T_{c2} \simeq 0.692J$. The intermediate phase is a pure chiral state in which only the Z_2 chirality, representing the direction of the \mathbf{l} -vector, aligns. In the low-temperature phase, in addition to the chiral long-range order, the $U(1)$ phase of the condensate exhibits the KT-type algebraic order leading to a true superfluidity. Meanwhile, the \mathbf{d} -vector representing the spin state of the condensate remains disordered at any finite temperature. Although in real films the dipolar interaction neglected here gives rise to additional interaction, it is several orders of magnitude weaker than J [2] and the ordering of the \mathbf{d} -vector, if any, should occur at a temperature much lower than T_{c1} and T_{c2} .

To identify the universality class of each transition, standard finite-size scaling analysis is made. The estimated chirality exponents at $T = T_{c1}$, $\nu = 1.0 \pm 0.1$ and $\beta/\nu = 0.13 \pm 0.01$, agree well with the values of the 2D Ising model, and I conclude that the upper transition belongs to the standard 2D Ising universality class. As regards the lower transition, the decay-exponent η_θ associated with the correlation function of the \mathbf{p}' -vector at $T = T_{c2}$, $\langle \mathbf{p}'(\mathbf{r}) \cdot \mathbf{p}'(\mathbf{0}) \rangle \approx r^{-\eta_\theta}$, is estimated from the log-log plot of $N < \mathbf{p}'^2 >$ versus L , yielding $\eta_\theta = 0.45 \pm 0.03$: See the inset of Fig2b. The estimated η_θ is considerably larger than the standard KT value, $\eta = 0.25$, suggesting that the lower transition is *not* of the standard KT universality. It should be noticed, however, that the spatial anisotropy neglected in deriving our model Hamiltonian (3) might possibly affect the value of η_θ .

In the present model, the intermediate phase is realized over an appreciable temperature range, in sharp contrast to the case of frustrated XY models where T_{c1} and T_{c2}

are identical or very close [11,12]. This difference is probably caused by the $SO(3)$ degree of freedom associated with the \mathbf{d} -vector. Namely, although the \mathbf{d} -vector itself does not order at any $T > 0$, its existence considerably affects the effective interaction between the chiral and phase variables. This may be seen by deriving an effective Hamiltonian \mathcal{H}_{eff} written in terms of the τ - and θ -variables, by integrating out the \mathbf{d} -variable. At sufficiently high temperatures, one can explicitly carry out this procedure to get

$$\mathcal{H}_{eff}/k_B T \approx - \sum_{\langle ij \rangle} \{ K_1 (1 + \tau_i \tau_j) \cos(2\theta_i - 2\theta_j) + K_2 \tau_i \tau_j \}, \quad (7)$$

where one has $K_1 = K_2$. Indeed, a numerical study in Ref.[14] suggested that in the case of $K_1 = K_2$ the model exhibited two successive transitions with an intermediate chiral phase consistent with the present result, whereas the frustrated XY models corresponded to $K_2 \simeq 0$ where the model (7) was estimated to exhibit a single transition only.

Finally, I discuss experimental implications of the obtained results. Main conclusion of the present work is that successive transitions and an intermediate phase are likely to be realized in thin films of ^3He . While the clearest sign of the upper transition is a logarithmically divergent anomaly in the specific heat, it might practically be difficult to get the necessary experimental sensitivity because specific-heat measurements usually require a considerable mass of sample. Some anomaly might be detectable at $T = T_{c1}$, however, in the quantities like thermal-expansivity, optical properties, and sound-velocity or attenuation (particularly, the third sound [15]). By contrast, the NMR shift, which has widely been used in identifying the bulk superfluidity transition, is blind to the distinction between 1 and -1 , and may not be a very good indicator of the transition. Rather, the shift should set in somewhat *above* T_{c1} , by an amount of order t^* , when the short-range order is developed and the \mathbf{l} -vector stays perpendicular to the surface.

Although the intermediate phase is *not* a true superfluid, it is expected to be a low-dissipation state distinguishable from the high-dissipation normal state above T_{c1} . Interestingly, the observation of a low-dissipation state was reported in the literature [5,6]. By contrast, the lower transition is characterized by the onset of a true superfluidity, and is detectable by the standard torsional-oscillator measurements. While the areal superfluid density ρ_s is expected to show a non-KT jump at $T = T_{c2}$,

$\rho_s/T_{c2} = 8m^2k_B/(\pi\eta_\theta\hbar^2)$ (m is the mass of ^3He atom), its absolute value $\rho_s/T_{c2} \sim 17.6 \times 10^{-12} \text{ g}/(\text{cm}^2\text{mK})$ is small reflecting the low T_c of ^3He , and experimental observation of the jump itself might be difficult.

Anyway, in the 2D superfluidity ordering of ^3He , new interesting physics different from that of helium four is expected. Further experimental studies are encouraged.

I thank K. Kono and K. Shirahama for useful discussions. The numerical calculation was performed on the FACOM VPP500 at the supercomputer center, ISSP, University of Tokyo.

- [1] J.M. Kosterlitz and D.J. Thouless, *J. Phys. C* **6**, 1181 (1973); D.R. Nelson and J.M. Kosterlitz, *Phys. Rev. Lett.* **39**, 1201 (1977).
- [2] A.J. Leggett, *Rev. Mod. Phys.* **47**, 1159 (1977).
- [3] D.R. Jones, A. Love and M.A. Moore, *J. Phys. C* **9**, 743 (1976).
- [4] One can roughly estimate t^* based on the Ginzburg criterion, $t^* \simeq [(2\pi\xi_0)^{-d}/\Delta C]^{2/(4-d)}$, where d is the dimensionality and ΔC is the specific-heat jump at T_c [*Modern Theory of Critical Phenomena*, S.K. Ma, Benjamin, p.94 (1976)]. If one puts $d = 2$ and substitutes typical values for the A-phase of ^3He , one gets $t^* \simeq 0.04$.
- [5] M.T. Manninen and J.P. Pekola, *J. Low Temp. Phys.* **52**, 497 (1983).
- [6] V. Kotsubo, K.D. Hahn and J.M. Parpia, *Phys. Rev. Lett.* **58**, 804 (1987).
- [7] J. Xu and C. Crooker, *Phys. Rev. Lett.* **65**, 3003 (1990).
- [8] M.R. Freeman and R.C. Richardson, *Phys. Rev. B* **41** (1990) 11011.
- [9] Y.-H. Li and T.-L. Ho, *Phys. Rev. B* **38** (1988) 2362.
- [10] D.L. Stein and M.C. Cross, *Phys. Rev. Lett.* **65**, 504 (1979).
- [11] S. Miyashita and H. Shiba, *J. Phys. Soc. Japan* **53**, 1145 (1984); D.H. Lee, J. D. Joannopoulos, J.D. Negele and D.P. Landau, *Phys. Rev. Lett.* **52**, 433 (1984).
- [12] S. Teitel and C. Jayaprakash, *Phys. Rev. B* **27**, 598 (1983); G. Ramirez-Santiago and J.V. José, *Phys. Rev. B* **49**, 9567 (1994); P. Olsson, *Phys. Rev. Lett.* **75**, 2758 (1995).
- [13] H. Kawamura, *J. Phys. Condens. Matter* **10**, 4707 (1998).
- [14] J. Lee, E. Granato and J.M. Kosterlitz, *Phys. Rev. B* **44**, 4819 (1991).
- [15] T. Ohmi, M. Nakahara, T. Tsuneto and T. Fujita, *Prog. Theor. Phys.* **68**, 1433 (1982).